

Problemas difíceis

$$1) \operatorname{tg} 2x + \operatorname{tg} x - \operatorname{tg} 3x = -\operatorname{tg} x \cdot \operatorname{tg} 2x \cdot \operatorname{tg} 3x$$

$$\operatorname{tg} x + \operatorname{tg} 2x - \operatorname{tg} (2x+x) = \operatorname{tg} x + \operatorname{tg} 2x - \frac{\operatorname{tg} 2x + \operatorname{tg} x}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} =$$

$$(\operatorname{tg} x + \operatorname{tg} 2x) \left[1 - \frac{1}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} \right] = (\operatorname{tg} x + \operatorname{tg} 2x) \left[\frac{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} - \frac{1}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} \right] =$$

$$\frac{(\operatorname{tg} x + \operatorname{tg} 2x) \cdot (-\operatorname{tg} 2x \cdot \operatorname{tg} x)}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} = \frac{-\operatorname{tg}^3 x \cdot \operatorname{tg} 2x - \operatorname{tg}^2 2x \cdot \operatorname{tg} x}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x}$$

$$= -\operatorname{tg} x \cdot \operatorname{tg} 2x \cdot \frac{(\operatorname{tg} x + \operatorname{tg} 2x)}{1 - \operatorname{tg} 2x \cdot \operatorname{tg} x} = \boxed{-\operatorname{tg} x \cdot \operatorname{tg} 2x \cdot \operatorname{tg} 3x}$$

$$2) \cos^2 \alpha + \cos^2 (\alpha + \beta) - 2 \cos \alpha \cdot \cos \beta \cdot \cos (\alpha + \beta) - \operatorname{sen}^2 \beta$$

$$\cos (\alpha + \beta) [\cos (\alpha + \beta) - 2 \cos \alpha \cdot \cos \beta] + \cos^2 \alpha =$$

$$\cos (\alpha + \beta) [-\operatorname{sen} \alpha \cdot \operatorname{sen} \beta - \cos \alpha \cdot \cos \beta] + \cos^2 \alpha =$$

$$\cos (\alpha + \beta) [-(\operatorname{sen} \alpha \cdot \operatorname{sen} \beta + \cos \alpha \cdot \cos \beta)] + \cos^2 \alpha =$$

$$- \cos (\alpha + \beta) \cdot \cos (\alpha - \beta) + \cos^2 \alpha =$$

$$= -[\cos \alpha \cdot \cos \beta - \operatorname{sen} \alpha \cdot \operatorname{sen} \beta] (\cos \alpha \cdot \cos \beta + \operatorname{sen} \alpha \cdot \operatorname{sen} \beta) + \cos^2 \alpha =$$

$$- [\cos^2 \alpha \cdot \cos^2 \beta - \operatorname{sen}^2 \alpha \cdot \operatorname{sen}^2 \beta] + \cos^2 \alpha =$$

$$= -[\cos^2 \alpha (1 - \operatorname{sen}^2 \beta) - (1 - \cos^2 \alpha) \operatorname{sen}^2 \beta] + \cos^2 \alpha =$$

$$- [\cos^2 \alpha - \cos^2 \alpha \cdot \operatorname{sen}^2 \beta - \operatorname{sen}^2 \beta + \cos^2 \alpha \cdot \operatorname{sen}^2 \beta] + \cos^2 \alpha =$$

$$- \cancel{\cos^2 \alpha} + \operatorname{sen}^2 \beta + \cancel{\cos^2 \alpha} = \operatorname{sen}^2 \beta$$