$$\begin{cases} x^{x+y} = y^{x-y} \\ x^2 \cdot y = 1 \end{cases}$$

$x^2 \cdot y = 1 \quad ; \quad y = \frac{1}{x^2} = x^{-2}$

$x^{x+y} = (x^{-2})^{x-y} \quad ; \quad x^{x+y} = x^{-2x+2y}$

$x + y = -2x + 2y$

$\boxed{3x = y}$

$$\begin{cases} y = 3x \\ x^2 \cdot y = 1 \end{cases}$$

$x^2 \cdot 3x = 1 \qquad 3x^3 = 1 \qquad x^3 = 1/3$

$x = \sqrt[3]{1/3} \qquad y = 3\sqrt[3]{1/3} = \sqrt[3]{\frac{3^3}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$

$x = \sqrt[3]{1/3} = \frac{1}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{\sqrt[3]{9}}{\sqrt[3]{3^3}} = \boxed{\frac{\sqrt[3]{9}}{3}}$

$\boxed{y = \sqrt[3]{9}}$

Jorge Cantabrana

$$\begin{cases} \log_5 x + 3^{\log_3 y} = 7 \\ x^y = 5^{12} \end{cases}$$

$3^{\log_3 y} = a$; $\log_3(3^{\log_3 y}) = \log_3 a$

$\log_3 y \cdot \log_3 3 = \log_3 a$

$\log_3 3 = b = 1$

$3^b = 3 \rightarrow b = 1$

$\log_3 y \cdot 1 = \log_3 a$

$\log_3 y = \log_3 a$

$\boxed{y = a}$

$$\begin{cases} \log_5 x + y = 7 \\ x^y = 5^{12} \end{cases}$$; $\log_5 x = 7 - y$; $5^{7-y} = x$

$(5^{7-y})^y = 5^{12}$; $5^{7y-y^2} = 5^{12} \rightarrow 7y - y^2 = 12$

$y^2 - 7y + 12 = 0$

$y = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm 1}{2} \begin{cases} 4 \\ 3 \end{cases}$

$\boxed{y = 4} \rightarrow x = 5^{7-y} = 5^3 = \boxed{125}$

$\boxed{y = 3} \rightarrow x = 5^{7-y} = 5^4 = \boxed{625}$